GLUON CONTRIBUTION TO THE TRANSVERSE SPIN STRUCTURE FUNCTION g_2

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Abstract

We consider the twist-3 gluon contribution to the transverse spin structure function g_2.We find that the Burkhardt-Cottingham sum rule is satisfied. The additional *nonlocal* operators appear in the moments of g_2 which were absent in previous analysis of Bukhvostov, Kuraev and Lipatov.

The only way we can get the information about the nucleon spin content in the polarized deep inelastic experiment is to measure the structure functions g_1 and g_2 which are the coefficients in the decomposition of imaginary part of the forward Compton ($T_{\alpha\beta}$) amplitude over appropriate tensor structures:

$$\begin{aligned} W_{[\alpha\beta]} &= \frac{1}{\pi} Im T_{[\alpha\beta]} \\ &= \frac{i}{(pq)} \epsilon_{\alpha\beta\rho\sigma} q_\rho [g_1(x_B,Q^2)s_\sigma + g_2(x_B,Q^2)\Big(s_\sigma - p_\sigma \frac{(sq)}{(pq)}\Big)] \end{aligned} \tag{1}$$

and

$$T_{\alpha\beta} = i\int d^4x e^{iqx} \langle p,s|T(j_\beta(x)j_\alpha(0))|p,s\rangle, \tag{2}$$

where s_α is a nucleon covariant polarization ($s^2 = -M^2$ for pure state, M is a nucleon mass), $x_B = \frac{Q^2}{2(pq)}$ is a Bjorken variable and q_ρ is four-momentum transfer.

There exist a set of sum rules for both g_1 and g_2. We remind the reader the Bjorken [1], Ellis-Jaffe [2] sum rules for the first one. The recent EMC result has put a great attention to the first moment of g_1 due to the large discrepancy with the naïve parton picture [3]. As for the second polarized structure function g_2 that measures the difference between the property of transversely and longitudinally polarized nucleon there exists the superconverging Burkhardt-Cottingham sum rule [4]:

$$\int_0^1 dx_B g_2(x_B) = 0. \tag{3}$$

There is some discussion in the literature concerning the validity of the last one [5]. This issue is made particularly important by near future measurement of g_2. Since the check of the Burkhardt-Cottingham sum rule is one of the planned experimental task it is essential to know whether the RHS of eq.(3) is expected to be nonzero.

The papers on higher twist effects in polarized deep inelastic scattering can be devided into three parts: the first one involves the works where the authors consider only flavour nonsinglet operators [6], in the second one the singlet channel is included but the triple gluon component is disregarded [7] and in the last one but not the least the evolution equations for partonic correlators is written where the twist-3 gluon operators appear in

the problem of mixing with the other operators under the renormalization group evolution [8]. So, the anomalous dimentions of gluon operators have already been found, however, their coefficient functions have not, though they look more important in practice. The gluons play the crucial role in the quark confinement and therefore we expect the 3-gluon effect to be large in the realistic nucleon. In this paper we investigate this question and present some physical implication of our results.

In the following discussion we will work in the light-cone gauge, $B_+ \sim (Bn) = 0$, where the vector n_μ can be constructed from the vectors of the problem: $n_\mu = \frac{1}{(pq)}[q_\mu + x_B p_\mu]$. One of the advantages of it is that one can easily recover gauge invariant expression after all required calculations have done.

The starting point of the analysis is the factorization procedure developed in the Ref. [9] which results in the separation of the hard coefficient function (E) from soft correlation function (N). On the first stage of factorization we obtain gauge variant formula for correlator in terms of gluon fields $\sim \langle B_\mu B_\nu B_\sigma \rangle$. Making use of the equation which directly connects gluon strength tensor and field itself (we drop the possible constant term due to the arbitrariness in the boundary condition):

$$B^a_\mu(\lambda n) = \frac{1}{2}\int_{-\infty}^{\infty} dz \epsilon(\lambda n^- - z) G^a_{+\mu}(z), \tag{4}$$

we acquire additional factor $[x_1 x_2 (x_2 - x_1)]^{-1}$ in the factorized expression for the structure function.

The three-gluon correlation function[1] can be decomposed into the two independent scalar functions time appropriate tensor structures [10]:

$$\begin{aligned} & N_{\mu\nu\sigma}(x_1, x_2) \\ = & -g(in^-)^3 \int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{-i\lambda x_2} e^{-i\mu(x_1 - x_2)} \langle p, s| i f^{abc} G^a_{+\mu}(0) G^b_{+\nu}(\mu n) G^c_{+\sigma}(\lambda n)|p, s\rangle \\ = & \, 2i[N(x_1, x_2) g_{\sigma\mu}\epsilon_{\nu pns} - N(x_1, x_1 - x_2) g_{\mu\nu}\epsilon_{\sigma pns} - N(x_2, x_2 - x_1) g_{\nu\sigma}\epsilon_{\mu pns} \\ & + \widetilde{N}(x_1, x_2)\epsilon_{\sigma\mu pn} s_\nu + \widetilde{N}(x_1, x_1 - x_2)\epsilon_{\mu\nu pn} s_\sigma - \widetilde{N}(x_2, x_2 - x_1)\epsilon_{\nu\sigma pn} s_\mu]. \end{aligned} \tag{5}$$

The hermiticity leads to the following symmetry properties of N and $\widetilde{N}$:

$$\begin{aligned} N(x_1, x_2) = N(x_2, x_1) \quad &, \quad N(x_1, x_2) = -N(-x_1, -x_2), \\ \widetilde{N}(x_1, x_2) = -\widetilde{N}(x_2, x_1) \quad &, \quad \widetilde{N}(x_1, x_2) = -\widetilde{N}(-x_1, -x_2). \end{aligned} \tag{6}$$

[1] Because of C-even photon state in the t-channel there appears only f-convolution of gluon fields.

An explicit form of N and $\widetilde{N}$ functions in terms of three-gluon correlators can be found by projecting the LHS of eq.(4) onto the tensor structures of the RHS. Solving the obtained six-by-six matrix equation with respect to $N(x_1, x_2)$ and $\widetilde{N}(x_1, x_2)$ we come to the expressions:

$$
\begin{aligned}
&2s_\perp^2 N(x_1, x_2)\\
=&-g(in^-)^3 is_\perp^\nu \int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{-i\lambda x_2} e^{-i\mu(x_1-x_2)} if^{abc}\langle p, s|\big(\tfrac{1}{2}G^a_{+\mu}(0)\widetilde{G}^b_{+\nu}(\mu n)G^c_{+\mu}(\lambda n)\\
&-\tfrac{1}{6}G^a_{+\mu}(0)G^b_{+\mu}(\mu n)\widetilde{G}^c_{+\nu}(\lambda n) - \tfrac{1}{6}\widetilde{G}^a_{+\nu}(0)G^b_{+\mu}(\mu n)G^c_{+\mu}(\lambda n)\\
&-\tfrac{1}{3}G^a_{+\mu}(0)\widetilde{G}^b_{+\mu}(\mu n)G^c_{+\nu}(\lambda n) + \tfrac{1}{3}G^a_{+\nu}(0)G^b_{+\mu}(\mu n)\widetilde{G}^c_{+\mu}(\lambda n)\big)|p, s\rangle \qquad (7)
\end{aligned}
$$

and

$$
\begin{aligned}
&2s_\perp^2 \widetilde{N}(x_1, x_2)\\
=&-g(in^-)^3 is_\perp^\nu \int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{-i\lambda x_2} e^{-i\mu(x_1-x_2)} if^{abc}\langle p, s|\big(\tfrac{1}{3}\widetilde{G}^a_{+\nu}(0)G^b_{+\mu}(\mu n)G^c_{+\mu}(\lambda n)\\
&-\tfrac{1}{3}G^a_{+\mu}(0)G^b_{+\mu}(\mu n)\widetilde{G}^c_{+\nu}(\lambda n) + \tfrac{1}{3}G^a_{+\mu}(0)G^b_{+\nu}(\mu n)\widetilde{G}^c_{+\mu}(\lambda n)\\
&+\tfrac{2}{3}G^a_{+\mu}(0)\widetilde{G}^b_{+\mu}(\mu n)G^c_{+\nu}(\lambda n) + \tfrac{2}{3}G^a_{+\nu}(0)G^b_{+\mu}(\mu n)\widetilde{G}^c_{+\mu}(\lambda n)\big)|p, s\rangle. \qquad (8)
\end{aligned}
$$

As can be easily understood the first line of eq.(5) contributes to the spin structure function g_2 while the second one gives rise to g_2 as well as to g_1. The last statement can be checked by explicit convolution of eq.(4) and the general tensor decomposition of the coefficient function:

$$
\begin{aligned}
t^{[\alpha\beta]\mu\nu\sigma} =& t_1^{(1)}\epsilon^{\alpha\beta q\nu}\epsilon^{\sigma\mu pq} + t_2^{(1)}\epsilon^{\alpha\beta q}{}_\rho g^{\sigma\mu}\epsilon^{\nu pq\rho} + t_3^{(1)}\epsilon^{\alpha\beta q}{}_\rho p^\nu \epsilon^{\sigma\mu q\rho}\\
&+\frac{1}{(pq)}t_4^{(1)}\epsilon^{\alpha\beta qp}q^\nu\epsilon^{\sigma\mu pq} + \frac{(pq)}{q^2}t_5^{(1)}\epsilon^{\alpha\beta q}{}_\rho q^\nu\epsilon^{\sigma\mu q\rho} + (\mu \to \nu \to \sigma \to \mu). \qquad (9)
\end{aligned}
$$

In order to find the coefficient function we have to evaluate the five-point Green function:

$$
\begin{aligned}
&t^{[\alpha\beta]\mu\nu\sigma}\\
=&\int d^4x e^{iqx}\langle 0|T(j_{[\beta}(x)j_{\alpha]}(0)if^{abc}B^a_\mu(x_1p)B^b_\nu((x_2-x_1)p)B^c_\sigma(-x_2p))|0\rangle_{amp}, \qquad (10)
\end{aligned}
$$

whose imaginary part gives us the coefficient functions of interest up to normalization factor that can be found by applying Wick theorem to the forward Compton amplitude. The relevant loop diagrams are shown in fig.1 (a,b).

The calculations are lengthy but straightforward and the obtained results are:

$$
\begin{aligned}
&g_2^{3gluon}(x_B) = 3\langle {Q_q}^2\rangle N_f \frac{\alpha_s}{4\pi} \\
&\times \int \frac{dx_1 dx_2}{x_1 x_2 (x_2 - x_1)} \left(E_2(x_1, x_2, x_B) N(x_1, x_2) + \widetilde{E}_2(x_1, x_2, x_B) \widetilde{N}(x_1, x_2) \right)
\end{aligned}
\tag{11}
$$

and

$$
g_1^{3gluon}(x_B) = 3\langle {Q_q}^2\rangle N_f \frac{\alpha_s}{4\pi} \int \frac{dx_1 dx_2}{x_1 x_2 (x_2 - x_1)} \widetilde{E}_1(x_1, x_2, x_B) \widetilde{N}(x_1, x_2), \tag{12}
$$

where $\langle {Q_q}^2\rangle = \sum \frac{Q_q^2}{N_f}$ is an average quark charge squared in the elementary units. Performing the calculation we have adopted the quark mass m_q as a IR-cutoff parameter:

$$
\begin{aligned}
&E_2(x_1, x_2, x_B) \\
&= \theta(x_B) \Big[\frac{3(x_2 + x_1)(x_B - x_2 + x_1)}{x_1 x_2 (x_2 - x_1)} \theta(x_2 - x_1 - x_B) \\
&- \frac{(x_B - x_2)(3x_2 - 2x_1)}{x_1 x_2^2} \theta(x_2 - x_B) \\
&- \frac{(x_B - x_1)(2x_2 - 3x_1)}{x_1^2 x_2} \theta(x_1 - x_B) \\
&- \frac{(x_2 + x_1)(x_B - x_2 + x_1)}{x_1 x_2 (x_2 - x_1)} ln(\frac{x_2 - x_1 - x_B}{x_B}) \theta(x_2 - x_1 - x_B) \\
&+ \frac{(x_2 - x_1)(x_B - x_2) - x_B x_1}{x_1 x_2^2} ln(\frac{x_2 - x_B}{x_B}) \theta(x_2 - x_B) \\
&+ \frac{(x_2 - x_1)(x_B - x_2) + x_B x_2}{x_1^2 x_2} ln(\frac{x_1 - x_B}{x_B}) \theta(x_1 - x_B) \\
&+ ln\left(\frac{m_q^2}{Q^2}\right) \Big(\frac{(x_2 + x_1)(x_B - x_2 + x_1)}{x_1 x_2 (x_2 - x_1)} \theta(x_2 - x_1 - x_B) \\
&- \frac{(x_2 - x_1)(x_B - x_2) - x_B x_1}{x_1 x_2^2} \theta(x_2 - x_B) \\
&- \frac{(x_2 - x_1)(x_B - x_2) + x_B x_2}{x_1^2 x_2} \theta(x_1 - x_B) \Big) + (x_i \to -x_i) \Big] \\
&+ (x_B \to -x_B)
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
&\widetilde{E}_2(x_1,x_2,x_B)\\
&=\theta(x_B)\Big[-\frac{(x_B-x_2+x_1)(3x_1^2+3x_2^2-2x_1x_2)}{x_1x_2(x_2-x_1)^2}\theta(x_2-x_1-x_B)\\
&-\frac{(x_B-x_2)[18x_1x_2-(x_2-x_1)(3x_2+2x_1)]+4x_Bx_1x_2}{x_1x_2^2(x_2-x_1)}\theta(x_2-x_B)\\
&+\frac{(x_B-x_1)[18x_1x_2+(x_2-x_1)(3x_1+2x_2)]+4x_Bx_1x_2}{x_2x_1^2(x_2-x_1)}\theta(x_1-x_B)\\
&+\frac{(x_B-x_2+x_1)(x_2^2+x_1^2)+2x_1x_2}{x_1x_2(x_2-x_1)}ln(\frac{x_2-x_1-x_B}{x_B})\theta(x_2-x_1-x_B)\\
&+\frac{(x_B-x_2)[6x_1x_2-(x_2-x_1)(x_2+x_1)]+x_Bx_1(3x_2+x_1)}{x_1x_2^2(x_2-x_1)}\\
&\qquad\qquad\qquad\qquad\times ln(\frac{x_2-x_B}{x_B})\theta(x_2-x_B)\\
&-\frac{(x_B-x_1)[6x_1x_2+(x_2-x_1)(x_2+x_1)]+x_Bx_2(3x_1+x_2)}{x_2x_1^2(x_2-x_1)}\\
&\qquad\qquad\qquad\qquad\times ln(\frac{x_1-x_B}{x_B})\theta(x_1-x_B)\\
&+ln\left(\frac{m_q^2}{Q^2}\right)\Big(-\frac{(x_B-x_2+x_1)(x_2^2+x_1^2)+2x_1x_2}{x_1x_2(x_2-x_1)}\theta(x_2-x_1-x_B)\\
&-\frac{(x_B-x_2)[6x_1x_2-(x_2-x_1)(x_2+x_1)]+x_Bx_1(3x_2+x_1)}{x_1x_2^2(x_2-x_1)}\theta(x_2-x_B)\\
&+\frac{(x_B-x_1)[6x_1x_2+(x_2-x_1)(x_2+x_1)]+x_Bx_2(3x_1+x_2)}{x_2x_1^2(x_2-x_1)}\theta(x_1-x_B)\Big)\\
&+(x_i\to -x_i)\Big]+(x_B\to -x_B). \qquad (14)
\end{aligned}
$$

The coefficient function which enter the polarized structure function g_1 is the following:

$$
\begin{aligned}
&\widetilde{E}_1(x_1,x_2,x_B)\\
&=\theta(x_B)\Big[-4\frac{4x_B-3x_2}{x_2(x_2-x_1)}\theta(x_2-x_B)+4\frac{4x_B-3x_1}{x_1(x_2-x_1)}\theta(x_1-x_B)\\
&+4\frac{2x_B-x_2}{x_2(x_2-x_1)}ln(\frac{x_2-x_B}{x_B})\theta(x_2-x_B)\\
&-4\frac{2x_B-x_1}{x_1(x_2-x_1)}ln(\frac{x_1-x_B}{x_B})\theta(x_1-x_B)
\end{aligned}
$$

$$+ln\left(\frac{m_q^2}{Q^2}\right)\Big(-4\frac{2x_B-x_2}{x_2(x_2-x_1)}\theta(x_2-x_B)+4\frac{2x_B-x_1}{x_1(x_2-x_1)}\theta(x_1-x_B)\Big)$$
$$+(x_i\to -x_i)\Big]+(x_B\to -x_B). \tag{15}$$

The terms in front of the IR logarithms is nothing but the leading logarithmic twist-three splitting functions.

We have finished with the calculation and now are in position to demonstrate some interesting consequences of these results. First of all, we find that the Burkhardt-Cottingham sum rule is satisfied supporting the well known result that in the light cone expansion there is no operator corresponding to the finite value of the first moment. However, this assertion corresponds to the contribution of local operators. In our approach there are operators that can enter the first moments of g_2 as well as g_1, they are of nonlocal origin: $\partial_+^{-1}G_{+\mu}\partial_+^{-1}G_{+\nu}\partial_+^{-1}\widetilde{G}_{+\mu}$ and $\partial_+^{-1}G_{+\mu}\partial_+^{-1}\widetilde{G}_{+\nu}\partial_+^{-1}G_{+\mu}$. But as follows from the eqs.(13)-(15) their Wilson coefficients vanish:

$$\int\limits_0^1 dx_B g_2^{3gluon}(x_B)=0. \tag{16}$$

Moreover

$$\int\limits_0^1 dx_B g_1^{3gluon}(x_B)=0. \tag{17}$$

This equation seems to be quite obvious because the nonzero value of the first moment of g_1 would lead to the violation of the Burkhardt-Cottingham sum rule. This observation follows from the facts arisen in the process of calculations. When we take the convolution of gluon legs indices with the tensor structure of $\widetilde{N}$-function in eq.(5) we extract the coefficients corresponding to the polarized structure function g_1 ($\widetilde{E}_1$) and to the sum g_1+g_2 ($\widetilde{E}_{1+2}$), not g_1 and g_2 separately. Therefore, the Burkhardt-Cottingham sum rule could not be violated for $M_1(g_1)\neq 0$ if the first moment of the difference $\widetilde{E}_{1+2}-\widetilde{E}_1$ equals zero. But we have the situation when the first moment of each term vanishes (eq.(17)).

A new feature appears in the third moments which was absent in the previous analysis of Bukhvostov, Kuraev and Lipatov: a set of nonlocal operators:

$$\int\limits_0^1 dx_B x_B^2 g_2^{3gluon}(x_B)$$

$$= \langle Q_q^2\rangle N_f \frac{\alpha_s}{4\pi}\Bigg[\left(\frac{5}{3}+\frac{2}{3}ln\left(\frac{m_q^2}{Q^2}\right)\right) 3\int dx_1 dx_2 \left(\frac{1}{x_1}+\frac{1}{x_2}\right) N(x_1,x_2)$$
$$+\left(-\frac{10}{3}-\frac{4}{3}ln\left(\frac{m_q^2}{Q^2}\right)\right) 6\int dx_1 dx_2 \frac{\widetilde{N}(x_1,x_2)}{x_2-x_1}$$
$$+\left(-\frac{15}{3}-\frac{6}{3}ln\left(\frac{m_q^2}{Q^2}\right)\right) 3\int dx_1 dx_2 \left(\frac{1}{x_1}-\frac{1}{x_2}\right) \widetilde{N}(x_1,x_2)\Bigg] \quad (18)$$

and

$$\int_0^1 dx_B x_B^2 g_1^{3gluon}(x_B)$$
$$= \langle Q_q^2\rangle N_f \frac{\alpha_s}{4\pi}\Bigg[\left(\frac{15}{3}+\frac{6}{3}ln\left(\frac{m_q^2}{Q^2}\right)\right) 6\int dx_1 dx_2 \frac{\widetilde{N}(x_1,x_2)}{x_2-x_1}$$
$$+\left(\frac{10}{3}+\frac{4}{3}ln\left(\frac{m_q^2}{Q^2}\right)\right) 3\int dx_1 dx_2 \left(\frac{1}{x_1}-\frac{1}{x_2}\right) \widetilde{N}(x_1,x_2)\Bigg]. \quad (19)$$

Where the integrals are expressed through the bilocal gluon operators:

$$3\int dx_1 dx_2 \left(\frac{1}{x_1}+\frac{1}{x_2}\right) 2s_\perp^2 N(x_1,x_2)$$
$$= -g(in^-)^3 s_\perp^\nu \int d\mu\epsilon(\mu) i f^{abc}\langle p,s|2G^a_{+\mu}(0)\widetilde{G}^b_{+\nu}(\mu n)G^c_{+\mu}(\mu n)$$
$$+\widetilde{G}^a_{+\mu}(0)G^b_{+\mu}(\mu n)G^c_{+\nu}(\mu n)+G^a_{+\nu}(0)G^b_{+\mu}(\mu n)\widetilde{G}^c_{+\mu}(\mu n)|p,s\rangle \quad (20)$$

and

$$6\int dx_1 dx_2 \frac{2s_\perp^2\widetilde{N}(x_1,x_2)}{x_2-x_1}$$
$$= -g(in^-)^3 s_\perp^\nu \int d\mu\epsilon(\mu) i f^{abc}\langle p,s|-2G^a_{+\mu}(0)\widetilde{G}^b_{+\nu}(\mu n)G^c_{+\mu}(\mu n)$$
$$-4\widetilde{G}^a_{+\mu}(0)G^b_{+\mu}(\mu n)G^c_{+\nu}(\mu n)-G^a_{+\nu}(0)G^b_{+\mu}(\mu n)\widetilde{G}^c_{+\mu}(\mu n)|p,s\rangle \quad (21)$$

and

$$3\int dx_1 dx_2 \left(\frac{1}{x_1}-\frac{1}{x_2}\right) 2s_\perp^2 \widetilde{N}(x_1,x_2)$$
$$= -g(in^-)^3 s_\perp^\nu \int d\mu\epsilon(\mu) i f^{abc}\langle p,s|G^a_{+\mu}(0)\widetilde{G}^b_{+\nu}(\mu n)G^c_{+\mu}(\mu n)$$
$$-\widetilde{G}^a_{+\mu}(0)G^b_{+\mu}(\mu n)G^c_{+\nu}(\mu n)+2G^a_{+\nu}(0)G^b_{+\mu}(\mu n)\widetilde{G}^c_{+\mu}(\mu n)|p,s\rangle. \quad (22)$$

Of cause we can easily restore the gauge invariant result substituting the path dependent exponents in the adjoint representation of the colour group into the bilocal operators.

For transversely polarized nucleon the combination $g_1 + g_2$ is measured experimentally:

$$\int_0^1 dx_B x_B^2 (g_1^{3gluon}(x_B) + g_2^{3gluon}(x_B))$$
$$= \langle Q_q^2 \rangle N_f \frac{\alpha_s}{4\pi} \left(\frac{5}{3} + \frac{2}{3} ln \left(\frac{m_q^2}{Q^2} \right) \right) \Big[3 \int dx_1 dx_2 \left(\frac{1}{x_1} + \frac{1}{x_2} \right) N(x_1, x_2)$$
$$+6 \int dx_1 dx_2 \frac{\widetilde{N}(x_1, x_2)}{x_2 - x_1} - 3 \int dx_1 dx_2 \left(\frac{1}{x_1} - \frac{1}{x_2} \right) \widetilde{N}(x_1, x_2) \Big]. \qquad (23)$$

So, we differ in a number of points with Ref. [8,**a**]. First, the evolution equation is written for the function $L(x_1, x_2)$ which is some combination of functions $N(x_1, x_2)$ and $\widetilde{N}(x_1, x_2)$:

$$L(x_1, x_2) = [2N(x_1, x_1 - x_2) + 2N(x_2, x_2 - x_1) -$$
$$\widetilde{N}(x_1, x_1 - x_2) - \widetilde{N}(x_2, x_2 - x_1)]. \qquad (24)$$

This equality holds up to the overall numerical constant which is not important for further discussion. So, we cannot express N and/or $\widetilde{N}$ separately through the function L, and therefore cannot compare the twist-3 splitting functions. Constructing a combination of L's with different arguments we come to the equation:

$$L(x_1, x_2) - L(x_1, x_1 - x_2) - L(x_2, x_2 - x_1)$$
$$= 4[N(x_1, x_2) - N(x_1, x_1 - x_2) - N(x_2, x_2 - x_1)], \qquad (25)$$

which could not resolve the above problems. This equality takes place because of similar symmetry properties of N and L with respect to permutation and change the sign of their arguments. But the most important supplement is the appearance of the *nonlocal* operators which are present not only in the third moment but also in the all highest moments of polarized structure functions. This become a common feature for triple gluon contribution as distinguished from the two gluon effect in g_1 widely discussed earlier where a nonlocal operator appears only in the first moment. But while in the last case it does not affect the evolution equation, the twist-3 evolution equations must be modified.

To summarise, we have calculated the coefficient functions of three-gluon correlators, and have discussed their physical consequences. The main results are:

(1) the Burkhardt-Cottingham sum rule is satisfied;

(2) the set of nonlocal operators appear in the moments of structure functions.

The last problem deserves further investigation and will be considered in the future publication.

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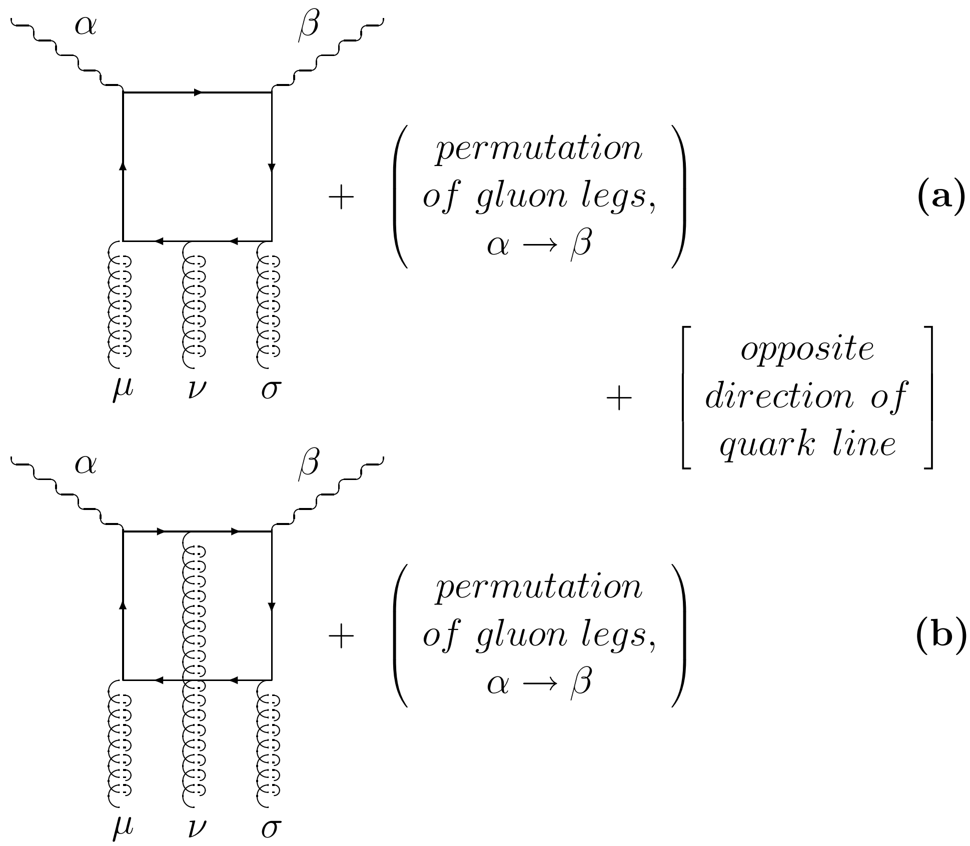

Figure 1: The lowest order diagrams for the coefficient function